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Stopping and filtering waves in phononic circuits

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Abstract

The acoustic band structures and transmissions through a one-dimensional (1D) monomode waveguide made of asymmetric slender tube loops pasted together with slender tubes of finite length are investigated theoretically. These monomode circuits may exhibit large stop bands where the propagation of acoustic waves is forbidden. These stop bands (gaps) originate both from the periodicity of the system and the resonant modes of the loops. The width of these bandgaps depends on the geometrical parameters of the structure and may be drastically increased in a tandem geometry made of several successive *asymmetric serial loop structures* (ASLSs) which differ in their *geometrical* characteristics. These ASLSs may have potential applications as ultra-wide-band filters.

The discovery of photonic crystals has laid the foundation of bandgap engineering in mesoscopic systems. The keynote behind the proposal of photonic crystals was the possibility of modifying the propagation of electromagnetic waves by creating photonic bandgaps in the band structure of synthetic periodic dielectric structures. Within a complete photonic bandgap, optical waves, spontaneous emission and zero-point fluctuations are all absent. Because of its promised ability to influence spontaneous emission [1], and to pave the way to light localization [2], the pursuit of photonic bandgaps has been the major motivation for studying photonic crystals. These materials are composed of periodically modulated dielectrics with the length scale of the periodicity approaching the wavelength of light. This constitutes an important part of mesoscopic physics, which is just beginning to be explored [3].

It did not take long before photonic crystals aroused interest in their phononic counterparts, namely, 'phononic crystals'—artificial two- and three-dimensional (3D) periodic elastic/acoustic composites [4]. In analogy to photonic crystals, the emphasis was laid on

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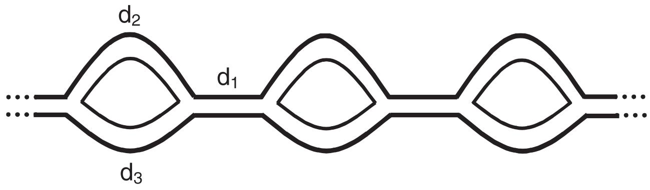

Figure 1. Schematic diagram of the one-dimensional asymmetric loop structure studied in the present work. The lengths of the three tubes are denoted d_1, d_2 and d_3 respectively.

the existence of complete elastic/acoustic bandgaps (ABGs) within which sound, vibrations and phonons are all forbidden. This is of interest for applications, such as elastic/acoustic filters and improvements in the design of transducers and noise control, as well as for pure physics concerned with the Anderson localization of sound and vibration [5]. Ferroelectric, pyroelectric, piezoelectric and piezomagnetic composites have had long standing applications as medical ultrasonic and naval transducers as well as for the related tasks in medical imaging [6]. Such composites were initially constructed for sonar applications and are now widely used for ultrasonic transducers.

Of particular interest is the existence of acoustic bandgaps in the band structure of one-dimensional (1D) structures with a variety of geometries [7–10]. In a previous paper [7], we demonstrated that the acoustic transmission spectrum of 1D comb structures exhibits large gaps. These structures, called *star waveguides*, are composed of N' dangling side branches (DSBs) periodically grafted at each of the N equidistant sites on slender tubes. The stop bands originate from the periodicity of the system determined by the distance between two neighbouring sites and from the eigen-modes of the DSB which play the role of resonators. The gap widths also depend on the boundary conditions at the free ends of the side branches, namely the open and closed tubes. In such systems the propagation is monomode provided that the two characteristic lengths (the periodicity and the resonator length) and the wavelength are much larger than the backbone and the side branch diameters [7]. These theoretical results are confirmed by experiments using an impulse response technique in the interval from 650 to 1100 Hz [9]. Unlike other 1D (e.g. Bragg lattices), 2D or 3D phononic crystals in which the contrast between the constituents is a critical parameter for the stop band existence, this *star waveguide* exhibits relatively large forbidden bands even if the backbone and the resonators are made of the same material [7].

The topic which will be addressed in this paper concerns the propagation of longitudinal (acoustic) waves through a quasi-one-dimensional structure, called an *asymmetric serial loop structure* (ASLS), of a monomode networked waveguide. The structure is composed of asymmetric slender tube loops pasted together with slender tubes of finite length (see figure 1). Such a structure may exhibit new features, in comparison with the star waveguide: for example, the existence of larger gaps, the avoidance of the constraint on the boundary condition at the end of the side branches, the appearance of quasi-quantized bands without inserting a defect and the achievement of complete gaps for a small number of loops. These new features (which could be of potential interest in acoustic waveguide structures) are essentially due to the asymmetry of the loop structure which is quite different from the case of the star waveguide [7]. We report on results of calculated band structures and transmission coefficients. We also show that the width of the bandgaps may be enlarged by coupling several ASLSs of different geometrical characteristics. Interestingly, we work in the framework of interface response theory (IRT) [11].

The 1D infinite ASLS can be idealized as an infinite number of unit cells pasted together. In each unit cell, the two arms of the ring have different lengths d_2 (of medium 2) and d_3

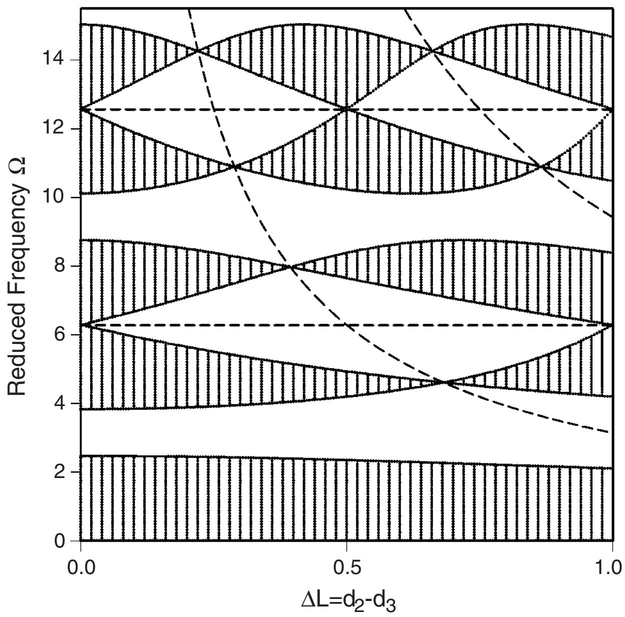

Figure 2. Projected band structure of the ASLS as a function of $\Delta L = d_2 - d_3$ for $d_1 = 0.5$ and $L = d_2 + d_3 = 1$. The shaded areas represent the bulk bands. The dashed curves indicate the frequencies for which the denominator of η (equation (1)) vanishes.

(of medium 3). This results in asymmetric loops of length $d_2 + d_3$ (see figure 1) which are pasted to slender tubes of length d_1 (of medium 1). We focus in this paper on homogeneous ASLSs where media 1, 2 and 3 are made of the same material. The dispersion relation of the infinite ASLS, that relates the pulsation frequency ω to the wavevector k, can be derived using the IRT [11]. It can be written as $\cos(kd) = \eta(\omega)$ where d stands virtually for the period of the structure and

$$\eta = \frac{1}{2\sin(\alpha(\omega)L/2)\cos(\alpha(\omega)\Delta L/2)}\left\{\sin(\alpha(\omega)d_1)\left[\frac{5}{4}\cos(\alpha(\omega)L) - \frac{1}{4}\cos(\alpha(\omega)\Delta L) - 1\right] + \cos(\alpha(\omega)d_1)\sin(\alpha(\omega)L)\right\}. \quad (1)$$

Here $L = d_2 + d_3$, $\Delta L = d_2 - d_3$ and $\alpha(\omega) = \omega/v$ where v is the longitudinal speed of sound.

Figure 2 displays the projected band structure (the plot is given as the reduced frequency $\Omega = \omega d_1/v$ versus ΔL) of an infinite ASLS for given values of L, and d_1 such that $L = 1$, and $d_1 = 0.5$ respectively. The shaded areas, corresponding to frequencies for which $|\eta| < 1$, represent bulk bands where acoustic waves are allowed to propagate in the structure. These areas are separated by mini-gaps within which the acoustic waves are forbidden. Inside these gaps, the dashed lines show the frequencies for which the denominator of η (equation (1)) vanishes. The dashed horizontal and curved lines, that correspond to the vanishing of $\sin(\alpha(\omega)L/2)$ and $\cos(\alpha(\omega)\Delta L/2)$ respectively, define the frequencies at which the transmission through a single asymmetric loop becomes exactly equal to zero. In figure 2, one can distinguish between two types of mini-gap: those of lozenge pattern that originate from the crossing of the zero-transmission lines, and the gaps around $\Omega = 3$ or 9 (occurring for any value of ΔL) that are related to the periodicity of the structure. There is one interesting point to notice in the band structure of figure 2; namely, at certain values of ΔL (for instance

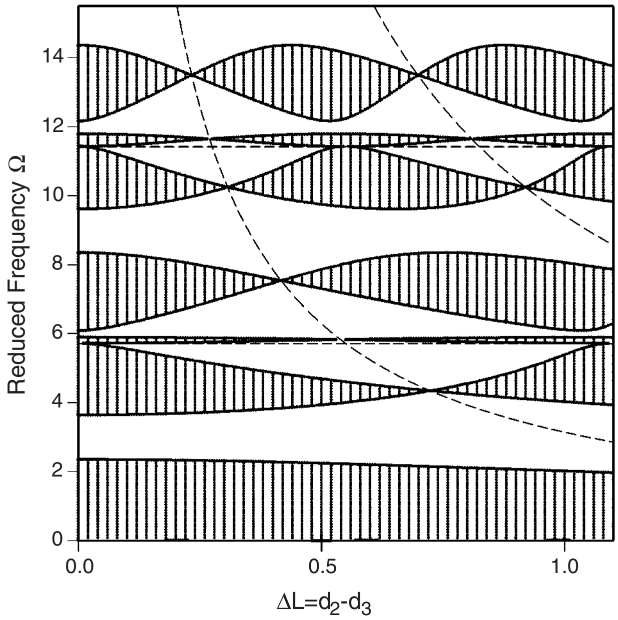

Figure 3. The same as in figure 2 but for an ASLS with $d_1 = 0.5$ and $L = d_2 + d_3 = 1.1$. Notice the existence of very narrow (almost flat) mini-bands created in the mini-gaps of lozenge pattern.

$\Delta L \sim 0.4$), one can obtain a series of narrow mini-bands separated by large gaps; this is because the points at which the mini-bands close align more or less vertically in such a way that a few successive bands may become very narrow.

Figure 3 displays the projected band structure for another geometry, namely, $d_1 = 0.5$ and $L = 1.1$. One can notice that some of the mini-gaps having a lozenge pattern display a large variation with ΔL. In the latter mini-gaps, one can observe the existence of very narrow (almost flat) mini-bands. These mini-bands become totally flat (and coincide with the horizontal dashed lines) if L is taken to be equal to 1 instead of 1.1. Indeed, for $L = 1$, we have $L = 2d_1$ and one can easily check that both the numerator and denominator of equation (1) vanish simultaneously at the frequencies of the dashed lines. The physical meaning of such flat bands is that the infinite structure possesses localized modes at these frequencies, while the transmission through the finite structure remains equal to zero. Let us finally mention that the existence and width of the mini-gaps are influenced by both the periodicity of the structure and the zeros of transmission.

We now turn to the study of the transmission probability. We start with a study of a simple example, namely a waveguide consisting of a unique asymmetric loop. The transmission coefficient T can be written as

$$T = \left| \frac{2(S_2 + S_3)S_2S_3}{(C_2S_3 + C_3S_2 + S_2S_3)^2 - (S_2 + S_3)^2} \right|^2, \tag{2}$$

where $C_i = \cosh[\alpha'(\omega)d_i]$, $S_i = \sinh[\alpha'(\omega)d_i]$, $\alpha'(\omega) = \mathrm{j}\alpha(\omega) = \mathrm{j}\omega/v$ and $\mathrm{j} = \sqrt{-1}$. The transmission is *equal to zero* only when $S_2 + S_3 = 0$, or equivalently, $2\sin[\alpha(\omega)L/2]\cos[\alpha(\omega)\Delta L/2] = 0$. Therefore, zeros of transmission coefficient occur at frequencies such that $\alpha(\omega)L = 2m\pi$ and $\alpha(\omega)\Delta L = (2m' + 1)\pi$ or equivalently

$$\omega_m = \frac{v}{\Delta L}(2m + 1)\pi, \tag{3}$$

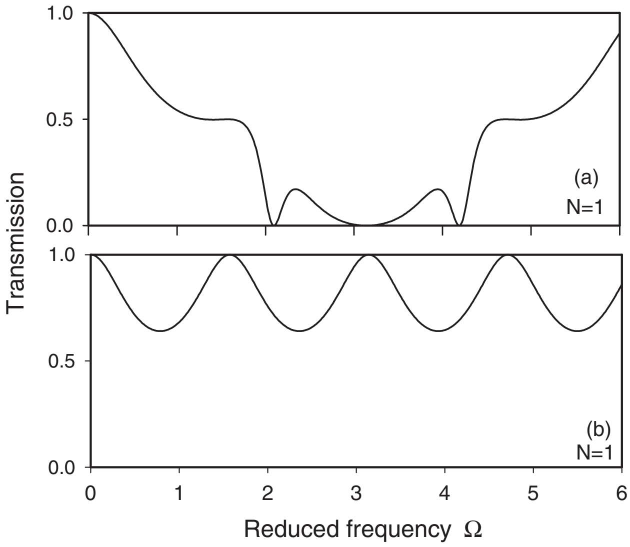

Figure 4. (a)Transmission factor versus the reduced frequency Ω for a waveguide with one loop in the case of an asymmetric tube loop with $L = 3$, $\Delta L = 1$. (b) The same as in (a) but for a symmetric tube loop ($d_2 = d_3$).

and

$$\omega_{m'} = \frac{v}{L} 2m'\pi, \tag{4}$$

where m and m' are integers.

It is worth noticing that for frequencies given by equation (3), the waves travelling on both paths of the loop are out of phase [12]. On the other hand, the frequencies given by equation (4) correspond to the eigen-modes of a loop alone.

The variations of the transmission coefficient T versus the reduced frequency Ω are reported in figure 4(a) for $\Delta L = 1$ and $L = 3$.

In the particular case of a symmetric loop ($d_2 = d_3$, i.e., $\Delta L = 0$), the transmission coefficient becomes

$$T = \frac{16}{25 - 9\cos^2(\alpha' d_2)}. \tag{5}$$

In contrast with the transmission coefficient of an asymmetric single loop, the transmission of a symmetric one never reaches zero values (see figure 4(b)). That is why, in symmetric serial loop structures, the gaps originate only from the periodicity. In contrast, in ASLSs, the gaps are due to the conjugate effect of the periodicity and the zero transmission associated with a single asymmetric loop which plays the role of a resonator.

In the case where the number of asymmetric loops becomes greater than one, the zeros of the transmission coefficient enlarge into gaps. Figure 5 illustrates the transmission spectrum versus reduced frequency Ω in ASLSs made up of one (5(a)), two (5(b)) and five (5(c)) asymmetric loops. The parameters are $d_1 = 1$, $L = 3$ and $\Delta L = 1$. The lowest panel (5(d)) shows the corresponding band structure. For $N = 2$ (figure 5(b)) we have only pseudo-gaps, not full gaps, in the system. As N increases the pseudo-gaps gradually turn into complete gaps (with transmission equal to zero) centred at almost the same mid-gap frequency.

The transmission rate through a finite-size ASLS containing $N = 10$ loops with $\Delta L = 0.4$, $L = 1$ and $d_1 = 0.5$ is reported in figure 6(a). Clearly, the existence of wide gaps separated

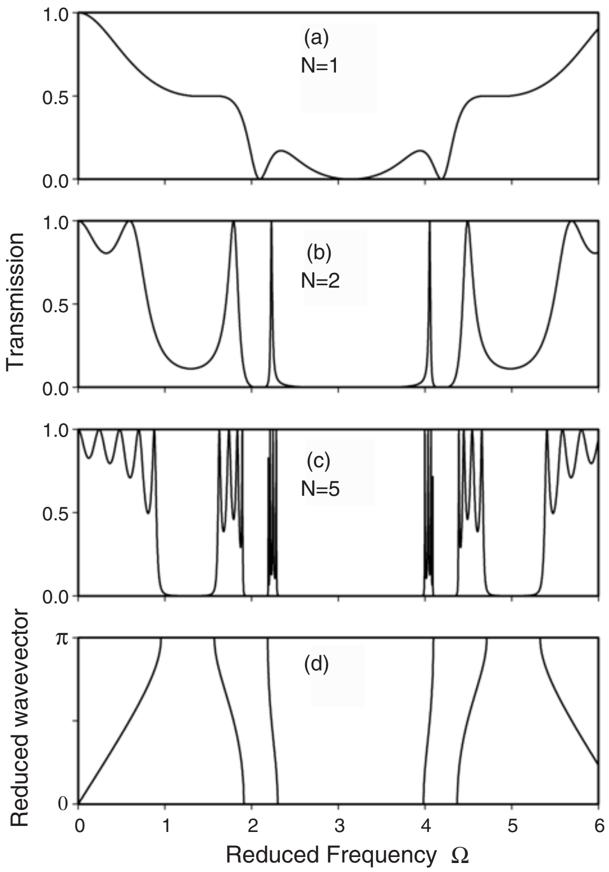

Figure 5. Evaluation of the transmission spectrum as a function of the number of loops N. The parameters are $d_1 = 1$, $L = 3$ and $\Delta L = 1$. Notice that as N increases the pseudo-gaps gradually turn into sharply defined complete gaps. The lowest panel shows the reflected band structure.

by narrow bands show up. Despite the finite number of loops in figure 6(a), the transmission approaches zero in regions corresponding to the observed gaps in the acoustic band structure of figure 1. It is worth noticing that the general features discussed in figure 1 are still valid for any values of d_1 and L and various ΔL. However, the shape of the band structure changes drastically for fixed values of d_1 and ΔL and various L. Figure 6(b) shows the transmission spectrum for another different ASLS with $N = 10$, $d_1 = 0.7$, $L = 1.4$ and $\Delta L = 0.4$.

Now, by associating in tandem the above ASLSs, one obtains (figure 6(c)) an ultra-wide gap where the transmission is cancelled over a large range of frequencies going from $\Omega \simeq 1.8$ to $\Omega \simeq 13.7$. In this structure, the huge gap results from the superposition of the forbidden bands of the individual ASLSs (figures 6(a) and (b)).

If a defect is included in the structure, a localized state can be created in the gap. A defect in ASLSs can be realized by replacing a finite wire of length d_1 by a segment of length $d_f \neq d_1$ in one cell of the waveguide. The transmission spectrum versus the reduced frequency for a structure with eight asymmetric loops, and a defect segment of length $d_f = 0.1d_1$ located in the middle of the waveguide, is depicted in figure 7. The frequency of the defect mode inside the gap depends on the length of the defect segment, whereas the intensity and the quality factor of the peak in the transmission spectrum depend on the number N of loops in the ASLS.

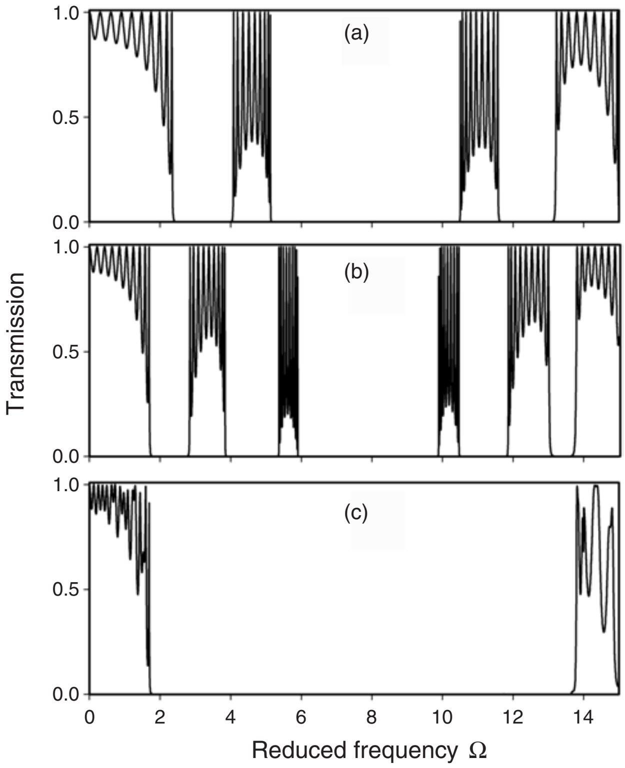

Figure 6. (a) Variations of the transmission power through an ASLS for $N = 10$ loops, $d_1 = 0.5$, $L = 1$ and $\Delta L = 0.4$. (b) The same as in (a), but for $d_1 = 0.7$, $L = 1.4$ and $\Delta L = 0.4$. (c) Transmission power through a tandem structure built of the above ASLSs (a) and (b). The superposition of the forbidden bands in (a) and (b) is clearly seen in (c).

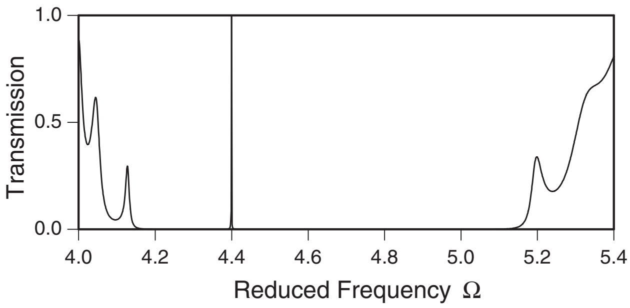

Figure 7. Transmission spectrum versus the reduced frequency Ω for an eight-loop ASLS with one defect segment of length $d_f = 0.1d_1$ located in the middle of the waveguide. The other parameters are considered to be $d_1 = 1$, $\Delta L = 1$ and $L = 2$.

In summary, we have investigated the existence of tunability of complete spectral gaps in the band structure of a quasi-one-dimensional waveguide with asymmetric slender loops pasted together with slender tubes of finite length. The waveguide segments and the loops can be made up of the same or different materials. Compared to other 1D networks such as the star waveguide [7], the observed gaps in the ASLS are significantly larger. The existence of the gaps in the spectrum is attributed to the conjugate effect of the periodicity and the zero transmission associated with a single asymmetric loop which plays the role of a resonator. In these systems, the gap width is controlled by the various parameters involved in the problem. The single symmetric defect is shown to introduce extra modes in the gaps of an otherwise periodic system. While the computation of the band structure requires an infinitely long periodic system, the transmission spectrum is calculated only for a finite system. The calculated transmission spectrum of acoustic waves in a finite ASLS parallels the band structure of the infinite periodic system. Accordingly, we conclude that the transmission spectrum in all cases remains consistent for five loops and more. We hope that these findings can be verified in an easily realizable set of experiments. Such systems can find some useful applications in the designing of transducers and ultrasonic filters.

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